

EXPERIMENTAL INVESTIGATION OF THE METHOD OF DETERMINING THE  
INTERNAL HEAT-TRANSFER COEFFICIENT IN A POROUS BODY FROM  
THE SOLUTION OF THE INVERSE PROBLEM

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The article presents the results of the practical application of a method of processing the data obtained in the course of full-scale thermophysical experiments.

The results of experimental investigations of the heat exchange inside porous bodies carried out by different authors show that there is considerable scatter of the experimental data, particularly, as was pointed out, e.g., in [1, 2], because the methods of determining the volumetric heat-exchange coefficients differ from each other.

The authors of [3, 4] dealt with the determination of the internal heat-transfer coefficient and of the effective thermal conductivity of a porous plate from the solution of the coefficient inverse problem based on the iteration algorithm of [5], with different conditions on its boundary surfaces.

To obtain a solution when boundary conditions of the first kind [3] are specified, it is indispensable to carry out temperature measurements inside the body; this entails the placing of temperature sensors which may to a considerable extent change the nature of the flow in the body, and in consequence distort in an unpredictable manner the process of heat transfer between the solid matrix and the gas blown through.

The authors of [4] dealt with the more general problem of finding the vector function  $\{T_S(x, \tau), \lambda_S(T_S), \alpha_V(\tau)\}$  from known nonsteady temperature measurements of the porous carcass at  $n$  points (including points on the boundary of the body) with specified initial temperature distributions for the solid and the gaseous phases, regularity of the change of coolant flow rate vs time, the hydraulic characteristics of the porous carcass and its volume heat capacity, and also the dependences of the thermophysical characteristics of the blown gas on the temperature. Here it was assumed that the process of heat exchange in the porous body is described by one-dimensional nonsteady equations of heat propagation in the porous carcass and in the coolant whose temperatures differ from each other. In the calculation of the thermal regime we adopted as boundary conditions: on the boundary which the coolant crosses, condition of the third kind; on the opposite boundary, condition of the second kind.

The algorithm for solving the coefficients of the inverse problem expounded in [4], together with the verification on model examples, was tested in the processing of the results of a real physical experiment. The values of  $\alpha_V$ , determined from temperature measurements on the boundary faces of the porous body in the process of unsteady cooling of the heated specimen, were compared with each other and with the values determined by traditional methods.

The investigated specimen, made from sintered stainless steel powder (with mean particle size 0.63 mm), was a disk with 50 mm diameter, 4 mm thick. The values of the coefficients of hydraulic resistance were  $\alpha = 2.3334 \cdot 10^{11}$  1/m<sup>2</sup> and  $\beta = 5.7267 \cdot 10^5$  1/m.

In the processing of the experimental data we endeavored to ensure the possibility of comparing the obtained values of  $\alpha_V$  with the values determined by traditional methods, and we therefore assumed that volume heat capacity and effective thermal conductivity of the carcass are specified exactly and have the following values:  $C_S = 0.5445 \cdot T_S + 1252.37$ , kJ/(m<sup>3</sup> · deg);  $\lambda_S = 4.5 \cdot 10^{-6} \cdot T_S + 2.92 \cdot 10^{-3}$ , kW/(m · deg). The thermophysical characteristics of the coolant were taken from [6].

The experiment was carried out in the following way. A radiative heating stream incident on the outer end face of the porous specimen, without the coolant being blown through, heated

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TABLE 1

No. of regime	$\dot{m} \cdot 10^3, \text{ kg/sec}$	$\alpha_v, \text{ kW}/(\text{m}^3 \cdot \text{deg})$			
		method of inverse problem			traditional method
		I	II	III	
1	2,506	424,0	197,2	291,4	1639,5
2	25,65	2668,0	725,1	1452,0	19892,1

**Note:** I) internal measurement; II) external measurement; III) internal and external measurements.

the specimen to  $\sim 750^\circ\text{K}$ ; after that, heating was discontinued and air was blown through. The temperature was measured on the outer and inner surfaces of the specimen. The duration of the experiment was determined by the specific flow rate  $\dot{m}$ . With the aid of the above-mentioned algorithm we processed the results of two regimes: with  $\dot{m} = 2.506 \cdot 10^{-3}$  and  $\dot{m} = 25.65 \cdot 10^{-3}$  kg/sec. The gas temperature at the inlet to the porous body was assumed to be constant during the experiment and equal to 294.16 and 288.91°K in each regime, respectively.

The temperature of the surfaces of the porous carcass was measured by Chromel-Alumel thermocouples with 0.2-mm diameter which were embedded by spot-welding. The coolant temperature at the inlet to the specimen was measured by a Chromel-Alumel antenna-type thermocouple with 0.2-mm diameter. The results were recorded by a loop oscillograph N115. The total error of temperature measurements and of the primary processing of the obtained results was estimated to be 5-6% of the maximal temperature.

The flow rate of coolant was determined from the pressure gradient on a measuring ring which was measured by a differential water manometer at air flow rates up to 0.005 kg/sec, and by a mercury manometer at higher flow rates. The accuracy of the measurements was  $\pm 4-5\%$ . Since there were no reliable data available on the value of the heat exchange coefficient at the inlet to the porous body, the inner surface of the porous body was assumed to be heat insulated ( $\alpha_0 = 0$ ). The internal heat transfer coefficient was determined for three cases where we took as initial experimental data the nonsteady temperatures of any one of the end-faces of the disk, or of both of them together.

The numerical solution of the boundary-value problems contained in the algorithm of identification was carried out on the space-time network  $\omega = \{i\Delta x, j\Delta \tau\}$ , where  $i = 1.30$ ;  $j = 1.32$ ;  $\Delta x = b/29$ ;  $\Delta \tau = \tau_m/31$ . These problems were approximated by a monotonic implicit difference scheme of second order of accuracy on the space coordinate and of first order of accuracy in time. The values of the internal heat-transfer coefficient found as a result of the calculations are presented in Table 1.

When we analyze the graphs in Fig. 1 we may conclude that not one of the variants of processing yields satisfactory agreement between the experimental and the calculated values of the temperatures of both surfaces simultaneously.

The deviation of the calculated temperatures from the experimental ones is due to: 1) insufficient accuracy of recording and interpreting the data of the measurements; 2) discrepancy between the real values of the initial and boundary conditions and of effective thermal conductivity and the values adopted in the calculations.

In addition to that, under real conditions there are some deviations from the adopted mathematical model of the heat-exchange process under consideration, specifically, the flow rate of the coolant does not remain constant. Because of the effect of compressibility, the flow rate of coolant increases gradually at the initial instant, and not jumpwise. Also the gas temperature at the inlet to the porous body is not strictly constant since at the initial instant the gas arriving at the inlet to the porous body is heated on account of heat conduction and natural convection to a temperature higher than  $T_{g_0}$  which was measured 40 mm upstream from the specimen. Finally, the condition of the inner surface of the specimen being thermally insulated is not strictly fulfilled.

The results of model calculations showed that the value of  $\alpha_v$ , obtained from the solution of the inverse problem, is affected most by the effective thermal conductivity and the heat-transfer coefficient at the inlet to the body.

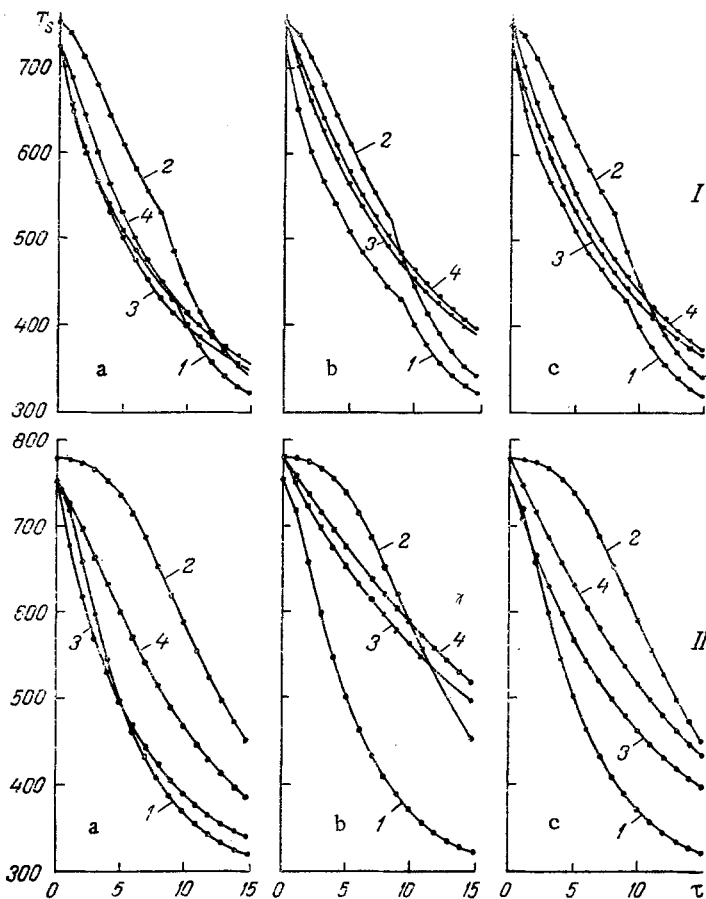


Fig. 1. Dependences of the temperatures of the outer and inner surfaces of the porous specimen: a) initial experimental data are the measurements on the inner surface; b) on the outer surface; c) on both surfaces; 1, 2) experimental curves; 3, 4) theoretical curves; I)  $\dot{m} = 2.506 \cdot 10^{-3}$  kg/sec, II)  $25.65 \cdot 10^{-3}$ ,  $\alpha_0 = 0$ .  $T_s$ , °K;  $\tau$ , sec.

Whereas for the determination of the effective thermal conductivity there is a sufficient amount of theoretical and experimental dependences (see, e.g., [7, 8]), which makes it possible to determine  $\lambda_s$  fairly accurately, for  $\alpha_0$  such dependences are not available. On the other hand, transition to nonzero value of  $\alpha_0$  has a substantial effect on the nature of the change of the temperature of the inner surface with time.

It follows from a comparison of Figs. 1b (II) and 2 that when the internal heat-transfer coefficient is determined from the solution of the inverse problem, heat exchange at the inlet has to be taken into account. If there is no reliable information on the values of  $\alpha_0$ , the coefficients  $\alpha_v$  and  $\alpha_0$  have to be determined simultaneously.

When two magnitudes are to be determined, the experimental dependences of the temperature at least at two points of the body have to be used to ensure uniqueness of the solution of the inverse problem. Therefore in distinction to the case of determining  $\alpha_v$  considered above, where three different variants of processing the experimental data are possible, only one variant was used in the simultaneous determination of  $\alpha_v$  and  $\alpha_0$ , where the data of the measurements on two boundary faces were taken into account. The corresponding results are presented in Figs. 3, 4. In solving the given problem, we took into account the constraints on the ranges of change of the sought magnitudes: the coefficients  $\alpha_v$  and  $\alpha_0$  cannot be smaller than zero, and the values of the heat-transfer coefficient at the inlet to the porous body must not exceed the maximally possible value determined from the condition  $T_s(0, \tau) \geq T_g(0, \tau)$ , i.e.,  $\alpha_0 \leq \rho v C_{pg}$ .

In case the sought values in any iteration exceeded the mentioned limits, we took as initial data for the subsequent iteration the values  $\alpha_v = 0.1$ ,  $\alpha_0 = 10^{-5}$  kW/(m<sup>2</sup>·deg) or  $\alpha_0 = \rho v C_{pg}$ , depending on which boundary the value of  $\alpha_0$  intersected.

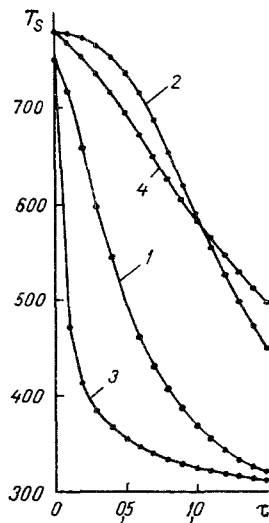


Fig. 2. Dependences of the temperatures of the outer and inner surfaces of the porous specimen in determining  $\alpha_V$  from measurement on the outer surface ( $\dot{m} = 25.65 \cdot 10^{-3}$  kg/sec,  $\alpha_0 = 2$  kW/(m<sup>2</sup>·deg); for 1-4, see Fig. 1.

The behavior of the process of iterative approximation with simultaneous determination of the constants  $\alpha_V$  and  $\alpha_0$  was affected greatly by the values of the initial approximations  $\alpha_V^0$  and  $\alpha_0^0$ . This is due to the complex nature of the dependence of the minimizing functional on the sought parameters. Figure 3 shows the isofunctional lines plotted according to the results of numerical calculations on the network  $\alpha_V = 0-300$  kW/(m<sup>3</sup>·deg) with a step of 10 kW/(m<sup>3</sup>·deg) and  $\alpha_0 = 0-1.8$  kW/(m<sup>2</sup>·deg) with a step of 0.1 kW/(m<sup>2</sup>·deg), and the trajectories of the motion from different initial approximations to the sought solution. In some cases, when the descent from the initial approximation was effected in a direction almost parallel to the "bottom of the ravine," the first step in the minimization of the functional ascertained internal heat transfer coefficients at (or close to) the limit values of  $\alpha_V = 0.1$  kW/(m<sup>3</sup>·deg), and further refinement was in fact carried out by one parameter. In that case the number of iterations was  $\sim 7-8$ . With other initial approximations we found a larger number of steps with the emergence in some practically constant direction of descent. Such behavior of the iteration process was found, e.g., with  $\alpha_V^0 = 300$  kW/(m<sup>3</sup>·deg) and  $\alpha_0^0 = 1.2$  or  $\alpha_0^0 = 1.3$  kW/(m<sup>2</sup>·deg) in descent by methods of conjugated gradients or of the steepest descent, respectively.

In some variants of processing we chose as initial approximations large values of the internal heat transfer coefficient ( $\alpha_V^0 = 2000-8000$  kW/(m<sup>3</sup>·deg)) with zero value of the heat transfer coefficient at the inlet. In these cases the iteration process led to values of  $\alpha_V$  and  $\alpha_0$  completely different from the previously obtained ones. To discover the causes of such behavior of the iteration process indicating bad conditionality of the inverse problem, we determined the values of the minimized discrepancy  $J(\alpha_V, \alpha_0 = 0)$  for a change of  $\alpha_V$  to 13,400 kW/(m<sup>3</sup>·deg) and a flow rate  $\dot{m} = 2.506 \cdot 10^{-3}$  kg/sec. The results of the calculations show that there are several local extrema along the axis  $\alpha_V$  which are fairly similar to each other in level. Thus, in dependence on the chosen initial approximation, different values of the heat-transfer coefficients  $\alpha_V$  and  $\alpha_0$  may be obtained.

In Fig. 4 we compare the theoretical dependences of the temperatures of the outer and inner surfaces with the extremally obtained values with different  $\alpha_V$  and  $\alpha_0$  ensuring the local minimum of the function  $J(\alpha_V, \alpha_0)$ . The best agreement between experiment and calculation is attained for  $\alpha_V = 0.1$  kW/(m<sup>3</sup>·deg) and  $\alpha_0 = 1.16$  kW/(m<sup>2</sup>·deg); however, more or less complete coincidence of the calculated and experimental values of the surface temperatures was thereby not obtained although the value of the minimized function is approximately half of the values obtained when only  $\alpha_V$  was sought on the assumption that the inner surface is heat insulated.

The obtained minimal values of the functional ((12) in [4]) correspond to the level of the total error of 3.5% of  $T_{max}$ , and they agree well with the results of the evaluation of the accuracy of measurements and of the primary processing.

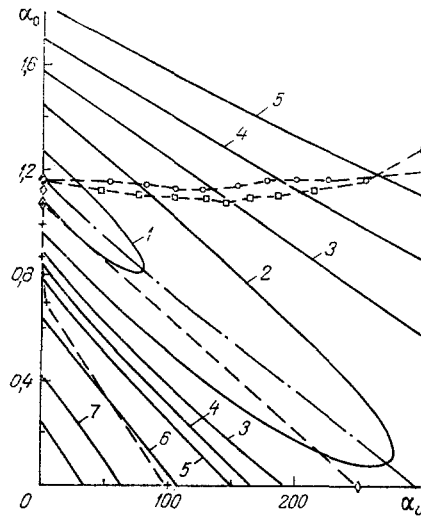


Fig. 3. Trajectory of the path of solution (dashed lines) with different initial approximations; solid curves are their iso-functional lines [1)  $J(\alpha_v, \alpha_0) = 2.5 \cdot 10^4$ ; 2)  $4 \cdot 10^4$ ; 3)  $6 \cdot 10^4$ ; 4)  $8 \cdot 10^4$ ; 5)  $10^5$ ; 6)  $5 \cdot 10^5$ ; 7)  $10^6$ ]; marks denote the points of the plane with the coordinates  $(\alpha_{vk}, \alpha_0^{(k)})$ , where  $k$  is the number of the iteration. The dot-dash line denotes the "bottom of the ravine."  $\alpha_0$ , kW/(m<sup>2</sup>·deg);  $\alpha_v$ , kW/(m<sup>3</sup>·deg).

As noted before, together with the heat transfer coefficients on the inlet to the porous body and inside it, the thermal regime is affected most by the effective heat transfer coefficient. Therefore any change of the values of  $\lambda_{sef}$  adopted in the solution of the inverse problem affects the obtained results. For instance, when the value of effective heat conduction is reduced by 10%, the functional attains its smallest value ( $J = 23,581$ ) with  $\alpha_v = 80.04$  kW/(m<sup>3</sup>·deg) and  $\alpha_0 = 0.8505$  kW/(m<sup>2</sup>·deg). When the values of  $\lambda_{sef}$  adopted in the processing of the experimental results are further reduced, the optimal values of  $\alpha_v$  increase, and those of  $\alpha_0$  decrease.

An analogous influence of the effective heat conduction was also found in the processing of the experimental data presented in [9]. Below we examine the results of this processing for two thermal regimes obtained in the investigation of a specimen 4.23 mm thick, with porosity 0.522, coefficients of hydraulic resistance  $\alpha = 1.545 \cdot 10^{12}$  1/m<sup>2</sup> and  $\beta = 2.522 \cdot 10^6$  1/m (specimen No. 8). The measured values were: flow rate intensity 0.22 and 0.523 kg/(m<sup>2</sup>·sec), wall temperature on the outer surface of the specimen 334.5 and 312.1°K, on the inner surface 325.5 and 308.6°K, initial temperature of the air blown through 292.5 and 296.9°K, air temperature at the outlet from the specimen 315.7 and 309.2°K for the first and second regimes, respectively.

The calculations were carried out on the assumption that the effective heat conduction of the porous carcass (whose values were not given in [9]) is determined by one of three dependences: Odelevskii's formula  $\lambda_{sef} = \lambda_M(1 - 1.5 \Pi)$ , Sherif's formula  $\lambda_{sef} = 0.5\lambda_M(1 - \Pi + (1 - \Pi)^2)$  [7], and by the dependence given above. The values of thermal conductivity of the material  $\lambda_M$  were specified on the basis of [10]. The thermal flux entering the porous body on its external side was adopted equal to

$$\lambda_s \frac{\partial T_s}{\partial y} = \rho v C_{pg} (T_g(b) - T_g(0)),$$

where  $T_g(0)$ ,  $T_g(b)$  are the measured gas temperatures at the inlet to and the outlet from the porous body, respectively.

As initial approximation of the sought values we took  $\alpha_v = 100$  kW/(m<sup>3</sup>·deg) and  $\alpha_0 = 0$  kW/(m<sup>2</sup>·deg). The results of the solution of the inverse problem, presented in Table 2, were obtained after ~18 iterations where the result was taken according to the condition of the closeness of the values of the functional in adjacent iterations.

With low flow rate intensity the optimal values of the heat transfer coefficient at the inlet and inside the porous body depend on the adopted values of effective thermal conduc-

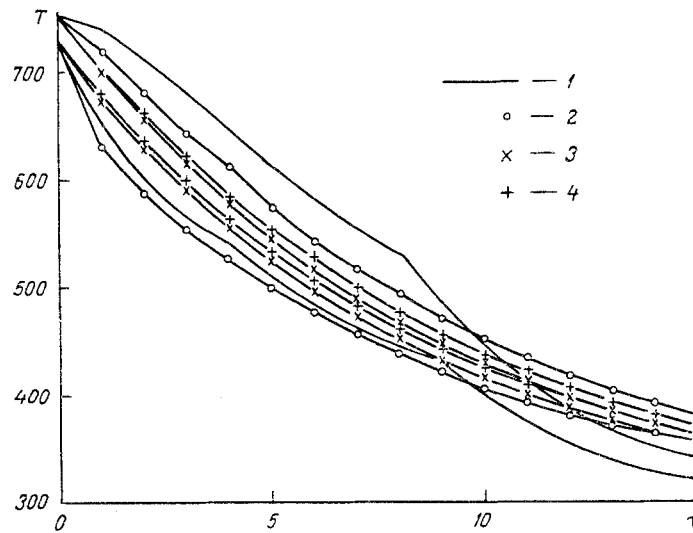


Fig. 4. Dependences of the temperatures of the outer (the upper curve from each pair of curves) and the inner (lower) surfaces of the porous specimen: 1) experimental curves; 2) calculated curves for  $\alpha_V = 0.1 \text{ kW}/(\text{m}^3 \cdot \text{deg})$  and  $\alpha_0 = 1.16 \text{ kW}/(\text{m}^2 \cdot \text{deg})$  ( $J = 21,725$ ); 3) 291.4 and 0 ( $J = 41,860$ ); 4) 5615.6 and 0 ( $J = 42,650$ ).  $T$ , °K.

TABLE 2

No. of regime	$\alpha_V$ from [9], $\text{kW}/\text{m}^3 \cdot \text{deg}$	$\lambda_s \text{ ef} = \lambda_M(1-1,5\Pi)$			$\lambda_s \text{ ef} = 4,5 \cdot 10^{-6} T_s + 0,00292$			$\lambda_s \text{ ef} = \lambda_M \frac{1-\Pi+(1-\Pi)^2}{2}$		
		$\alpha_V$	$\alpha_0$	F	$\alpha_V$	$\alpha_0$	F	$\alpha_V$	$\alpha_0$	F
1	50,0	11,067	0,1474	0,1252	105,986	0,391	2,2258	138,062	0,50366	5,113
2	241,9	150,0	$1 \cdot 10^{-6}$	39,694	153,006	$1 \cdot 10^{-6}$	9,492	136,875	$1 \cdot 10^{-6}$	0,895

tivity. This entails considerable heat removal at the inlet to the porous body which in [11, 12] was ascribed to internal heat exchange; as a result of this in the criterial equations  $\text{Nu} = f(\text{Re})$  there appeared the factor  $d_4/h$  ( $d_4$  is the particle diameter;  $h$  is the thickness of the specimen) which contradicts the physical meaning.

With high flow rate intensity (regime 2) and for all values of  $\lambda_{s \text{ ef}}$  the obtained values of  $\alpha_V$  and  $\alpha_0$  differ little from each other, and the contribution of  $\alpha_0$  to the heat balance is negligibly small. The calculated values of the temperature of the porous carcass with  $\alpha_V = 90\text{-}250 \text{ kW}/(\text{m}^3 \cdot \text{deg})$  and  $\alpha_0 = 0 \text{ kW}/(\text{m}^2 \cdot \text{deg})$  lie in the range of permissible temperatures of the porous carcass measured with an accuracy of 2-3% [9].

Our investigation showed that the determination of the internal heat transfer coefficient in a porous body has to be carried out with a view to the heat exchange at the inlet to the porous body or the heat transfer coefficients have to be determined simultaneously at the inlet to the porous body and inside it. To improve the accuracy of determining heat transfer coefficients, it is necessary to endeavor to improve the measuring techniques and to reduce the systematic errors. It is also indispensable to plan the experiment beforehand, i.e., to determine the regimes of executing the experiment and the thicknesses of the specimen with which the experimental investigations are carried out in the range of the highest sensitivity of the minimizing function to changes in the sought values.

#### NOTATION

$x$ , coordinate;  $b$ , thickness of the porous body;  $n$ , number of measurements of the temperature of the porous body;  $C_S$ , volume heat capacity;  $\lambda_S$ , effective thermal conductivity of the porous body;  $\rho$ ,  $C_p$ ,  $\lambda$ ,  $\mu$ , density, specific heat, thermal conductivity and viscosity of the blown gas;  $T$ , temperature;  $\alpha_V$ ,  $\alpha_0$ , heat-transfer coefficients inside the porous body and at the inlet to it, respectively;  $\rho_V$ , intensity of blowing through;  $\tau$ , time;  $\tau_m$ , duration of the experiment;  $\alpha$ ,  $\beta$ , coefficients of hydraulic resistance of the porous plate;  $\Pi$ , porosity;  $q$ , heat flux to the wall on the outer boundary;  $\dot{m}$ , coolant flow rate per second;  $\psi$ ,  $\varphi$ , conjugated variables;  $q_V$ , intensity of internal heat release. Subscripts:  $s$ , solid phase;  $g$ , gaseous phase;  $wa$ , wall.

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### DETERMINATION OF HEAT-TRANSFER COEFFICIENTS AT THE INLET INTO A POROUS BODY AND INSIDE IT BY SOLVING THE INVERSE PROBLEM

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An iterative algorithm is developed of searching these coefficients from data of nonstationary temperature measurements.

A large number of experimental studies is devoted to the study of features of internal heat transfer. These studies are divided in [1] into two groups, depending on the method of determining the internal heat-transfer coefficient. A characteristic feature of most of the studies considered in [1] is the use of the assumption of negligibly small heat transfer at the inlet to a porous wall, which, as noted in [2], must lead to an enhanced experimentally determined  $\alpha_v$  value in comparison with the true one.

The real pattern of heat transfer at the inlet into a porous body can be described by means of a boundary condition of third kind, used in [3], where an algorithm is provided for determining the coefficients of internal heat transfer and effective thermal conductivity of a porous plate by solving the inverse problem.

The practical use of this algorithm is rendered difficult in several cases due to the absence of verifiable information on values of the heat-transfer coefficient at the inlet into the plate. Therefore, in searching  $\alpha_v$  from temperature measurements, for example, obtained in the process of nonstationary cooling of a sample heated by gas blowing, it is advisable to determine simultaneously the heat-transfer coefficients at the inlet into a porous body and inside it, as well as the effective thermal conductivity coefficient of a porous housing under conditions guaranteeing unique solution of the problem.

In the present study we consider an algorithm of simultaneous search of  $\alpha_v$  and  $\alpha_0$  under the assumption that the  $\lambda_{\text{seff}}$  values are given accurately. The basic reason for this restriction is the complexity, as well as the impossibility of placing a thermocouple inside the porous structure due to the breakdown in the character of cooler filtration. Therefore, in most experimental investigations it is only possible to place two thermocouples at the surface boundaries of the plate. Under these conditions it is not possible to determine simultaneously all the heat-transfer characteristics mentioned above, and the original problem must be decomposed into several stages, such as an initial search of the effective thermal conductivity of a porous housing (for which one can use the algorithms derived in [3, 4]), and

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